Exercise 1: Harvest models

Consider a population of fish with dynamics according to logistic growth. We want to use this population as a resource and we are looking for a harvesting strategy that guarantees large and stable yield. We consider the following two strategies:

A. With constant-rate harvesting (e.g., because of fishing quota), we have

$$\dot{N} = rN\Big(1 - \frac{N}{K}\Big) - H\,. \tag{1}$$

B. With relative-rate harvesting, we will catch fish proportional to the stock size,

$$\dot{N} = rN\Big(1 - \frac{N}{K}\Big) - EN, \tag{2}$$

where E measures the fishing effort (this could be a quota on fishing boats).

We define the *maximum sustainable yield* (MSY) as the largest yield that can be taken from the species' stock over an infinite period.

1. Make a bifurcation analysis for harvesting strategy A with H as parameter (diagrams and formulas for bifurcation points). What kinds of bifurcations occur?

2. Make a bifurcation analysis for harvesting strategy B with E as parameter (diagrams and formulas for bifurcation points). What kinds of bifurcations occur?

3. What is the MSY for both strategies and for which parameter values of H and E do we get this yield? How do these values relate to the bifurcation points? What implications for harvesting strategies do you see? Which strategy should be preferred?

4. Assume that the fish population exhibts a *strong Allee effect* and thus cannot grow at low densities. This can be modeled, for example, by an extra factor $(K_0/K)(N/K_0 - 1)$, $K_0 \ll K$, multiplied to the logistic growth term in Eqs. (1) and (2). How do the bifurcation diagrams change for both strategies? Does this change your conclusions?