Exercise 1: Harvest models

Consider a population of fish with dynamics according to logistic growth. We want to use this population as a resource and we are looking for a harvesting strategy that guarantees large and stable yield. We consider the following two strategies:

1. With constant-rate harvesting (e.g., because of fishing quota), we have

$$\dot{N} = rN\Big(1 - \frac{N}{K}\Big) - H\,. \tag{1}$$

2. With relative-rate harvesting, we will catch fish proportional to the stock size,

$$\dot{N} = rN\Big(1 - \frac{N}{K}\Big) - EN, \tag{2}$$

   where E measures the fishing effort (this could be a quota on fishing boats).

We define the *maximum sustainable yield* (MSY) as the largest yield that can be taken from the species' stock over an infinite period.

1a/2a) Make a bifurcation analysis for both harvesting strategies with H and E as parameters (diagrams and formulas for bifurcation points). What kinds of bifurcations occur?

1b/2b) What is the MSY and for which parameter values of H and E do we get this yield? Which implications for harvesting strategies do you see? Which strategy should be chosen?